Cut-off free finite zero-point vacuum energy and the cosmological missing mass problem

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ABSTRACT

As the mass-energy is universally self-gravitating, the gravitational binding energy must be subtracted self-consistently from its bare mass value so as to give the physical gravitational mass. Such a self-consistent gravitational self-energy correction can be made non-perturbatively by the use of a gravitational 'charging' technique, where we calculate the incremental change dm of the physical mass of the cosmological object, of size r_o due to the *accretion* of a bare mass dM, corresponding to the gravitational coupling-in of the successive zero-point vacuum modes, i.e., of the Casimir energy, whose bare value $\Sigma_{\mathbf{k}}\hbar ck$ is infinite. Integrating the 'charging' equation, $dm = dM - (3\alpha/5)Gm\Delta M/r_o c^2$, we get a gravitational mass for the cosmological object that remains finite even in the limit of the infinite zero-point vacuum energy, i.e., without any ultraviolet cut-off imposed. Here α is a geometrical factor of order unity. Also, setting $r_o = c/H$, the Hubble length, we get the corresponding cosmological density parameter $\Omega \simeq 1$, without any adjustable parameter. The cosmological significance of this finite and unique contribution of the otherwise infinite zero-point vacuum energy to the density parameter can hardly be overstated.

Key words: vacuum energy, zero-point energy, missing mass, cosmology, Casimir effect.

1   INTRODUCTION

The single most important quantity determining cosmological evolution is the average mass density (ρ) of the universe, usually re-expressed as the dimensionless density parameter $\Omega = \rho/\rho_c$, where $\rho_c \equiv 3H^2/8\pi G$ is the critical mass density required to spatially close the universe. Here H is the Hubble *constant*, the other observationally most important expansion parameter. Thus, with $H = 100 h km^{-1}s^{-1}Mpc^{-1}$, we have $\rho_c = 2.10$ x $10^{-29}h^2 g$ cm^{-3}, with h of order unity. It has been known for sometime now that the visible baryonic matter contents of a typical spiral galaxy hardly add up to about one tenth of the actual mass of the galaxy, estimated dynamically from the rotation curve of its outer spiral arms. Other cosmological considerations suggest that the mass of galaxies, including that of the invisible matter, in which the galaxy seems embedded, in turn con-

stitutes a mere one fifth of the closure mass density of the universe. Thus most of the gravitational mass of the universe is missing. This missing-mass problem has forced cosmologists to propose the existence of a variety of non-baryonic dark matter (see Sciama, 1993) − the Cold Dark Matter (CDM), the Hot Dark Matter (HDM) along with various candidate particles, e.g., neutrinos, axions and other exotic extended objects such as cosmic domain walls and cosmic strings, that bear on structure formation. Recent developments in cosmology involving in particular the high red-shift observations at accurately known cosmological distances, made possible by the discovery of Type Ia supernovae serving as *standard candles*, strongly favour $h = 0.65$, and, most importantly, a density parameter value $\Omega = 1$ (for a critical discussion on this, see Coles, 1998). Such a critical choice is also indicated from considerations of the inflationary Big Bang models, and not a little from the criteria of aesthetics and naturalness. Normally $\Omega = 1$ would imply a spatially flat, ever expanding and ever decelerating Friedmann universe. But recent observations also suggest a possibly accelerated expansion (see Coles, 1999). This has forced cosmologists to re-introduce Einstein's cosmological constant Λ, the third most important parameter now (See Peebles, 1999). This has been invoked as an *X-matter*, and observations support $\Omega = \Omega_\Lambda + \Omega_m = 1$ with $\Omega_\Lambda = 0.7$, and $\Omega_m = 0.3$, the remaining mass density including dark matter. Now, the cosmological constant is quintessentially a vacuum energy, accumulated from the successive first-order phase transitions that the inflationary universe is believed to have undergone following the hot Big Bang. The high energy calculations for this quintessential matter density ($\Omega_\Lambda = \frac{\Lambda}{3H^2}$), however, give a value much too large, a factor $\sim 10^{120}$ times greater than the total density of all the matter in the universe. Thus, Ω, H and Λ continue to remain three free parameters of modern cosmology − getting continually constrained by observations, but uncertainty prevails still. Be that as it may, it is a compelling thought among cosmologists now that the universe does seem, after all, pervaded by some kind of vacuum energy that accounts for the missing mass. In this note we address this problem, and examine the possibility of this vacuum energy being none other than the familiar zero-point energy of the physical vacuum associated with some quantized Bosonic field, e.g., the photonic vacuum, and show that its otherwise infinite and, therefore, worrisome value gets renormalized self-gravitationally down, to yield a cosmological density parameter Ω which is not only finite, but in fact close to unity, and essentially free of any adjustable parameter. We are talking here of a cosmological Casimir Effect resolving the missing mass problem!

A cosmological role for the zero-point energy has been considered in

the past (Kumar, 1969a, 1969b; see also Mostenpanenko and Trunov, 1988; and Milonni, 1994). In order to fully appreciate the point of this proposal, let us recall that the bare zero-point energy of the electromagnetic vacuum $E_{ZP} = \Sigma_{\mathbf{k}} \hbar ck$ is obviously infinite. The cut-off wavelength $\lambda_c = 2\pi/k_c$ required to give an $\Omega \simeq 1$ is $\lambda_c \simeq 0.2$ mm which is astrophysically completely unacceptable on grounds of local Lorentz invariance (Wesson, 1991). A cut-off of $\lambda_c \sim \ell_{planck} \equiv (\hbar G/c^3)^{1/2}$ while acceptable astrophysically would give a density parameter which is practically infinite. While this infinity can be, and is, subtracted away through a re-definition of the zero of energy, or equivalently by normal ordering as is usually done in quantum-field theory, no such subtraction is permissible here as the non-linearly self-coupled gravitational effect must be absolute, and it is large. But the effect is, however, conspicuous by its absence here. Indeed, the very reality of the zero-point energy has been doubted for this very reason (see Milonni, 1994. But, the zero-point energy is fundamental, arising due to the non-commutativity of the electric and the magnetic field operators (in principle much the same way as the zero-point energy of the condensed matter,e.g., of the quantum liquid ^{4}He, is due to the non-commutativity of the atomic position and the corresponding momentum operators. Of course, in case of the condensed matter it is possible to directly measure its contributions to the cohesive energy inasmuch as the condensed phase can be assembled from the dispersed gas phase). Then there is the well known laboratory evidence for its local effect − the Casimir force between plates of a capacitor due to the change of boundary conditions and the associated vacuum-mode *depletion*. It is, therefore, most compelling to accept the reality of the zero-point energy. The problem really is with its being infinite! It has been suggested that this infinity can be cancelled exactly by invoking supersymmetry, but the latter is clearly badly broken in nature. The only way out now is to show that this *bare-ly* infinite zero-point energy gets gravitationally self-renormalized to a finite value which is acceptable cosmologically. This indeed turns out to be the case as we will show now.

2   DERIVATION

Consider the cosmological object (the universe) of size r_o having initially a gravitational mass $m = m_o$ (of presumably baryonic origin) in the absence of any gravitational coupling to the zero-point modes of the physical vacuum. We will now follow the gravitational *charging* of this object by gravitationally coupling in the zero-point modes incrementally. Let at some stage of the charging the current mass m of the object be incremented by Δm due to the coupling in, or *accretion*, ΔM from the vacuum modes. We

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have then the *charging* equation

$$\Delta m = \Delta M - (6\alpha/5)Gm\Delta M/r_o c^2. \tag{1}$$

It is essential here to realize that this *charging* procedure is for a fixed r_o. Here α is a geometrical factor of O(1), its Euclidean value being 1. The equation (1) can be readily integrated to give a renormalized (gravitationally self-energy corrected) mass m for the object:

$$\ell n \frac{(1 - \frac{6\alpha}{5}Gm_o/r_o c^2)}{(1 - \frac{6\alpha}{5}Gm/r_o c^2)} = \frac{6\alpha}{5} \frac{GM}{r_o c^2}. \tag{2}$$

¿From Eq. (2) we readily see that as the *accreted* gravitational *charge m* increases from zero to the full bare value of the zero-point energy (which is infinite), the gravitational mass m of the cosmological object increases from m_o to $m(\infty) = (\frac{5}{6\alpha})\frac{r_o c^2}{G}$.. This renormalized value corresponds to a mass density $\rho = (\frac{5}{2\alpha\beta})\frac{c^2}{4\pi r_o^2}$, where β is another geometrical factor of 0(1), its Euclidean value being 1. Taking the size of the object to be $r_o = c/H$ the Hubble length, we get for the density parameter

$$\Omega = \frac{\rho}{\rho_c} = \frac{5}{3}(\frac{1}{\alpha\beta}). \tag{3}$$

This is our main result.

3   DISCUSSION AND CONCLUSIONS

From Eq. (3), $\Omega = 5/3$ for $\alpha = \beta = 1$, their Euclidean value. Thus, we get the renormalized cosmological mass density not only finite, but also the corresponding value of the density parameter Ω turns out to be close to unity! It is to be emphasized here that inasmuch as all zero-point energies must gravitate universally, the above result is quite independent of the particular quantized field, e.g., the photons here, with which the zero-point energy is associated. Also, it should be remarked here that this global (cosmological) effect in no way alters the local (laboratory) Casimir effect that involves local changes of the boundary conditions for the quantized modes, e.g., at the metallic plates of the capacitor.

It is apt to contrast here our gravitational *charging* procedure for the cosmological object as in Eq. (1) with the one that underlies the ADM mass (Arnowitt *et al.*, 1960) of a self-gravitating non-cosmological object placed in an asymptotically flat space, namely

$$dm = dM - (6/5)Gmdm/c^2 r_o. \tag{4}$$

Notice the subtle difference between Eqs. (1) and (4), in that our Eq. (1) contains ΔM, and not Δm, in the second term on the RHS. Our charging procedure is consistent with the physical idea that adding on a bare mass dM to the pre-existing gravitational mass m should give a gravitational mass-defect proportional to mdM. Equation (4) can be readily integrated to give the well-known ADM mass m, that tends to ∞ as M tends to infinity. This difference is remarkable and calls for further examination.

Finally, it is to be noted that we have considered here only the overall cosmological consequence of the zero-point energy. Its association with the baryonic matter, important for structure formation, has not been touched upon.

In conclusion we have shown that the zero-point energy of the vacuum − the Casimir energy − is gravitationally self-renormalized to yield a finite value for the density parameter Ω which turns out to be close to unity, without any adjustable parameter such as the ultraviolet cut-off. This gravitational self-energy correction is carried out self-consistently through a *charging* procedure. This is a step forward towards resolution of the missing mass (Dark Matter) problem in cosmology. One could perhaps say that the Dark Matter is after all made of Light − the zero-point photon energy!

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